# Extraction of Magneto-Dielectric Properties from Metal-Backed Free-Space Reflectivity

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***Abstract*—Intrinsic magnetic and dielectric properties of magneto-dielectric composites are typically determined at microwave frequencies with both transmission and reflection data. An iterative method, such as root-finding, is often used to extract the properties in a frequency-by-frequency basis. In some situations, materials may be manufactured on a metal substrate that prevents transmission data from being obtained. This happens when the materials are too fragile or too strongly bonded to the substrate for removal and must be characterized with the metal substrate in place. This paper compares two different free-space extraction algorithms, developed for the simultaneous extraction of complex permittivity and permeability from metal-backed reflection. One of the algorithms relies on reflection measurements of the same material with two known thicknesses. The second method takes advantage of wide bandwidth measurements to fit the reflection to analytical models (e.g. Debye). The accuracy of these methods are evaluated and the stability criteria for the techniques will be discussed, as well as the tolerance of the techniques to various measurement errors.**

## I. INTRODUCTION

Dielectric permittivity and magnetic permeability are complex values, and four measured parameters (or two complex ones) are needed to fully determine an isotropic material's electromagnetic properties. One method to determine these properties is through the measurement of network scattering parameters from a slab of material. These can be collected through waveguide-based or free-space measurements of complex reflection and transmission for each frequency. An inversion algorithm then relates these measured scattering parameters to the desired intrinsic properties.

For some materials, transmission and reflection may not be available or may be inconvenient to collect. This paper considers the case of materials that are backed by a metal surface, such as a coating applied onto a metal plate, which is not easily removed from the metal substrate after application. In such cases, the collection of transmission values is not possible due to the metal backing, and all materials parameters must be determined through the reflection parameters. In the special case of materials with already known permeability (e.g. dielectrics with an assumed $\mu \sim 1$), analytic solutions exist to obtain values for the materials permittivity. In cases with unknown values for both permittivity ($\varepsilon$) and permeability ($\mu$), however, no solution exists to obtain these parameters from a single measurement.

In this paper, we explore two strategies to obtain both permittivity and permeability from metal-backed magneto-dielectric materials. The first method acquires two separate measurements from samples of the same material with different thickness. In this case, we solve an analytic equation to extract the material properties. In our second method, we apply a dispersion model to the material, and extract the properties of the material from the fitting parameters. The advantage of this second case is that material parameters are determined from a single measurement. Both methods have shortcomings, which are analyzed in this paper.

## II. EXPERIMENTAL DETAILS

The algorithms developed in this paper were tested with a tabletop free-space measurement system shown in Figure 1. This system uses two spot probes connected to a two-port network analyzer. The probe performance was previously documented to have accuracy similar to a larger laboratory focused beam system [1]. A commercial magneto-dielectric absorber was used to test the developed algorithms. It is a carbonyl iron filled elastomer obtained from Arc Technologies, specifically their 11802 magnetic material. The thicknesses of these specimens were measured as 75 mils (1.9 mm)

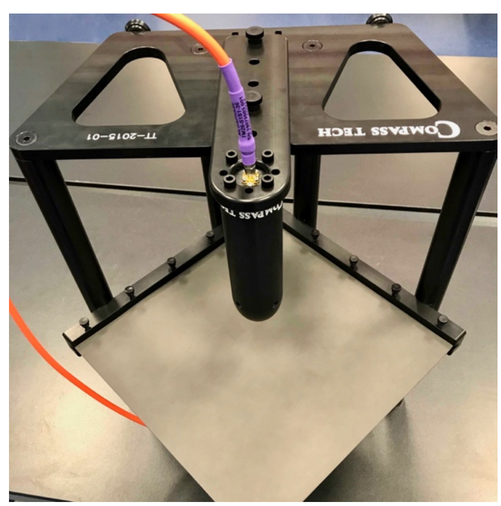

**Figure 1 Photo of tabletop free space measurement.**

## III. Two-Thickness Inversion

The two-thickness inversion model is derived starting with the equation for reflection from a metal-backed sample:

$$S_{11} = \frac{\mu \tanh(itk_0\sqrt{\mu\varepsilon})-\sqrt{\mu\varepsilon}}{\mu \tanh(itk_0\sqrt{\mu\varepsilon})+\sqrt{\mu\varepsilon}} \qquad (1)$$

Here, the free-space wavevector and sample thickness are represented as $k_0$ and t, respectively. After some rearrangement, and noting that sqrt(με) is the refractive index, n, of the material and that sqrt(μ/ε) is the impedance, $Z_m$ of the material, we obtain the apparent normalized impedance of the material:

$$\frac{1-S_{11}}{1+S_{11}} = Z = Z_m tanh(itk_0 n) \qquad (2)$$

Since the material's impedance does not vary with thickness, we rearrange the above equation to solve for $Z_m$, and then set these equations equal to each other. (Note that the subscripts 1 and 2 are used to distinguish between the two samples)

$$\frac{Z_1}{tanh(it_1 k_0 n)} = \frac{Z_2}{tanh(it_2 k_0 n)} \qquad (3)$$

This equation is solved for refractive index with a numerical root finder (e.g. Newton's Method), allowing for an explicit solution to equation 2, which provides permittivity and permeability values. One key advantage of this method is that materials properties theoretically can be obtained for any frequency without prior knowledge of the material under study.

To test the algorithm, we obtained permittivity and permeability data for a magnetic absorber sample using a more conventional S11/S21 inversion method [2]. With these material parameters, we used equation 1 to generate simulated reflection (S11) expected for metal backed samples of this same magnetic material for two different thicknesses. The fitted metal-backed S11 data for two different thicknesses (1 mm and 3 mm) were then used to test the two-thickness inversion. Figure 2 plots the results of this second inversion, showing that the fitted permittivity and permeability match almost exactly with the measured values. This case shows that the algorithm can handle real material parameters. However, the reflection produced from these simulated materials is idealized and does not necessarily capture the uncertainty introduced in actual measurements.

To obtain directly measured data, two magnetic absorber sheets were measured first separately (1.9 mm), and then stacked on top of each other to create a thicker specimen (3.8 mm). Care was taken to minimize air gaps between the two sheet specimens. The experimentally measured sheets consisted of a carbonyl iron powder mixed into an elastomer via mechanical milling. As is typical for this manufacturing method, the magnetic absorber had a small anisotropy – the permittivity and permeability in one direction was different versus the other. Thus, the orientation of each sheet was chosen to ensure that the material properties along the polarization direction of the measurement fixture were most consistent between the two measurements.

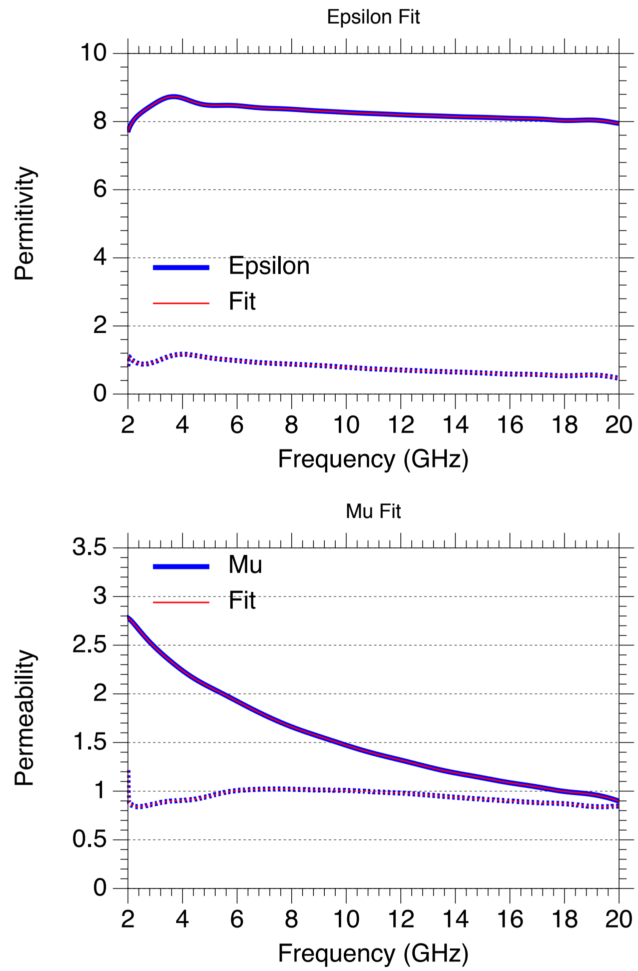


**Figure 2 Two-thickness algorithm fit to simulated metal-backed S11 data of a commercial magnetic absorber compared to conventional free-film inverted properties.**

Applying the two-thickness inversion algorithm to the data yielded the results in Figure 3. Both the two-thickness inverted data (thin lines) and conventional free-film (transmission and reflection) inverted data (thick lines) are shown in these plots. Overall, the agreement of the two-thickness data is close to the results obtained with transmission/reflection-based inversion. Some of the differences can be explained by accuracy limitations, particularly the poor fit to permittivity near the lower end of the frequency band. The accuracy of the two-thickness method at low frequencies for dielectric properties is limited by the weak tangential electric field that exists near a metal boundary, i.e. for a metal-backed specimen. Another possible source of error lies within the material itself. Since the two-thickness method requires two separate measurements of different specimens, differences with the materials properties between the two specimens will increase measurement uncertainty.

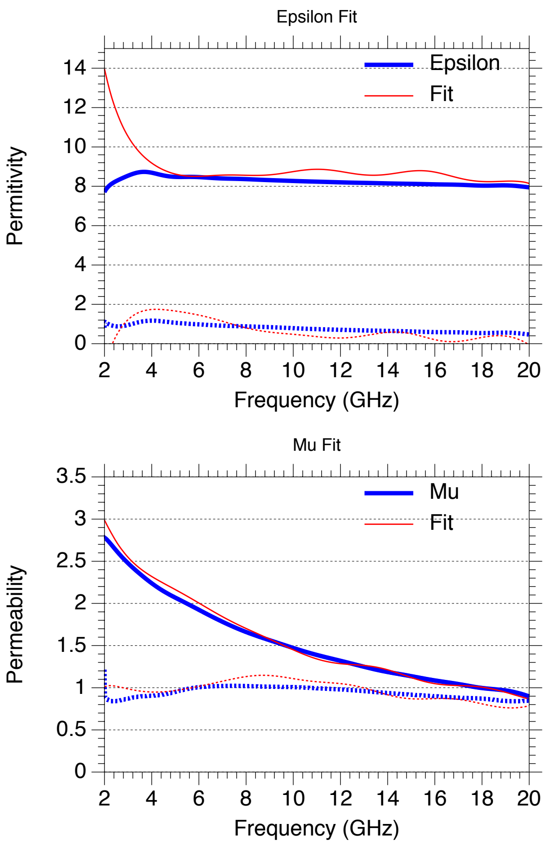


**Figure 3 Two-thickness algorithm fit to directly measured metal-backed S11 data from two different thicknesses of a commercial magnetic absorber compared to conventional free-film measurement.**

A limitation to the two-thickness inversion method is that equation 3 has multiple solutions due to the periodic nature of the phase shift in the material. Practically, this requires a reasonable initial guess for the material properties to converge onto the correct solution, as with many other material inversion algorithms. Figure 4 shows the solution space of the two-thickness inversion for the measured data above. The top plot shows the two-layer model fit 'goodness' at 3 GHz, while the bottom plot shows the same model fit 'goodness', but at 17 GHz. This goodness was calculated as the difference between measured data and the two-thickness model equation for each combination of real and imaginary index ($n$ = sqrt($\mu\varepsilon$)). As shown in these plots, the number of solutions increases with increasing source frequency (and electrical thickness) so a useful method to ensure the correct solution is to solve for low frequency data first and use these obtained values to inform guesses for higher frequency data.

The two-thickness inversion method implicitly assumes that the materials under study are identical materials with the sole exception of a difference in thickness. In practice, it may not always be possible to ensure that the two specimens have identical properties (e.g. due to variations in manufacturing). Since, strictly speaking, there will always be some nonzero differences in properties, it is important to understand how this inversion method will respond to such uncertainty. To take an extreme case, we simulated a material so that the permittivity doubled between layers. The results are shown in Figure 5. The fitting of the real part of $n$ shows a transition in the middle of the frequency band from a higher value to a lower value. These values correspond to the two indices of the thicker and thinner samples. The imaginary part shows an anomalous peak above 2, whereas both materials individually have imaginary index below 0.5 at all frequencies. This error is not simply a numerical error within the algorithm, as the fitted value corresponds to the 'correct' best solution within the solution space of equation 3. This demonstrates that the material properties between the two measured samples should match as closely as possible to ensure meaningful results.

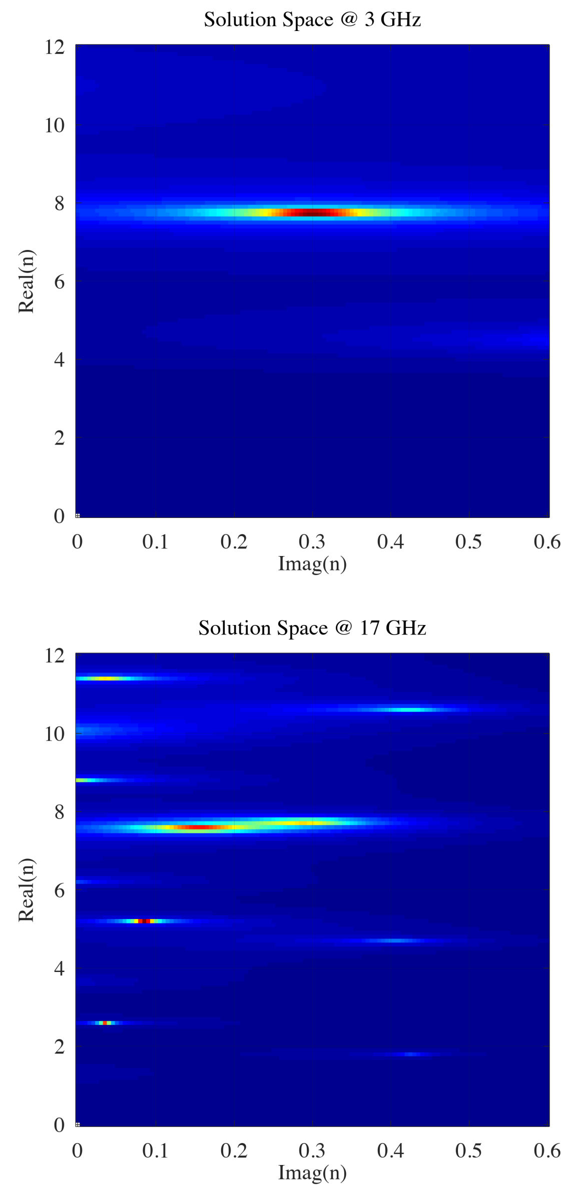


**Figure 4 The two-thickness model 'goodness' of fit at low frequency (top) and high frequency (bottom). Multiple maxima (light blue to red areas) are evident in the higher frequency solution space.**

$$\varepsilon = \varepsilon_{inf} + \frac{\varepsilon_s - \varepsilon_{inf}}{1 + i\frac{\omega}{\omega_\varepsilon}} \quad (4)$$

$$\mu = \mu_{inf} + \frac{\mu_s - \mu_{inf}}{1 + i\frac{\omega}{\omega_\mu}} \quad (5)$$

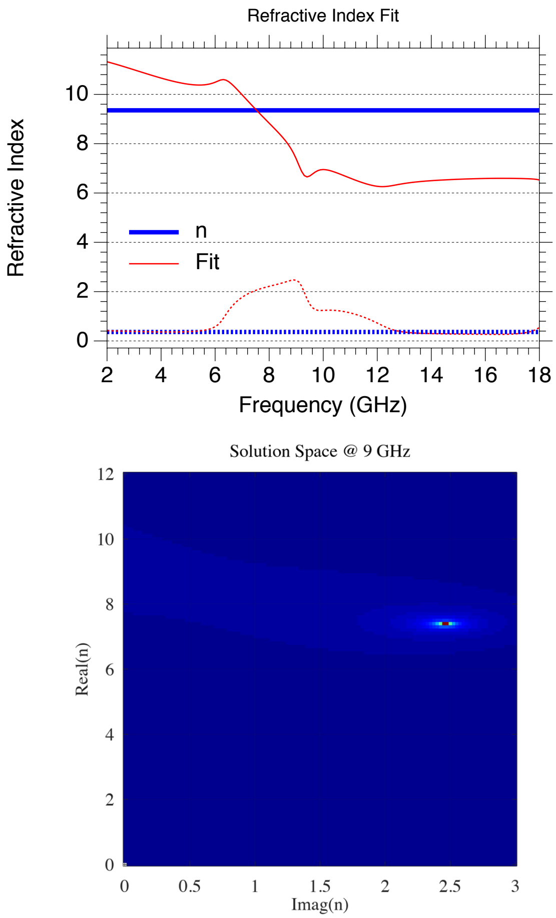


**Figure 5 An example of the anomalous dispersion calculated by the two-thickness inversion when the two thickness specimens have significantly different properties.**

## IV. Debye Model Inversion

While the two-thickness method is effective for determining intrinsic properties of metal-backed magneto-dielectric materials, it has the disadvantage of requiring two different material samples. A more desirable situation is where the complex permittivity and permeability can be obtained from a single sample of metal-backed material. However, solving for four unknown parameters (real and imaginary permittivity and permeability) based on two measured values (reflection amplitude and phase) is obviously not practical for frequency-by-frequency inversion. Instead, the model-based approach to metal-backed inversions uses the broadband nature of free-space measurements to fit assumed model parameters to the observed S11 data. In other words, rather than trying to fit four intrinsic parameters at each frequency, the model-based algorithm fits a six or so model parameters across all the frequencies in the measured bandwidth. In this method and to keep the method practical, we employ Debye models [3] for both the permittivity and the permeability of the material.

This Debye formulation has three fitted model parameters for permittivity and another three for permeability leading to six fitting parameters ($\varepsilon_0, \varepsilon_\infty, \mu_0, \mu_\infty, \omega_\varepsilon, \omega_\mu$) and a dependence on wavenumber ($\omega$). This formulation is approximate, and it is certainly possible to use more complex functions (such as Lorentz). However, adding more parameters will increase computation time and complexity and it is not clear that there is a benefit in accuracy to using a more complicated dispersion model. That said, it is not uncommon for some materials to have a characteristic conductivity in the dielectric properties. In that case a conductivity term may be added to the permittivity model to account for that conductive material behavior. For the magnetic absorber examples shown in this paper however, an additional conductivity term is not required.

Figure 6 shows an example of an ideal material with permittivity described by a Debye model. Over a wide frequency band, a Debye material's real permittivity transitions from the low frequency limit to the high frequency limit. During this transition, the imaginary part of the permittivity reaches a peak. For practical materials measurements, this transition will typically not span more than a decade of frequencies, leading to variable effects depending on where in the Debye relaxation the measurement band takes place. In the high or low frequency limits, the Debye material is a non-dispersive, low-loss material, whereas the dispersion and loss become significant near the relaxation frequency.

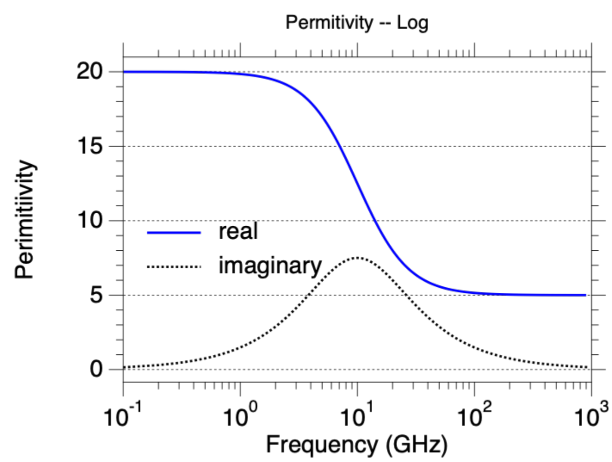


**Figure 6 Ideal material described by the Debye dispersion; versus logarithm of frequency (top) and linear frequency in the 2-18 GHz band (bottom)**

As shown in **Error! Reference source not found.**, a change in the Debye parameters will cause a resulting change in the reflection of the material. Typically, a large number of frequency points is collected while the number of fitting parameters is 6. After obtaining the fitting parameters, the material values are easily obtained with the above equations. One obvious deficiency of this method is that the model fit is not guaranteed to converge onto a solution if the underlying material properties are not well described by a simple Debye model. Similar to the two-thickness method, this method also requires a reasonable initial guess for convergence of the model, even for materials that are perfectly described by the Debye model.

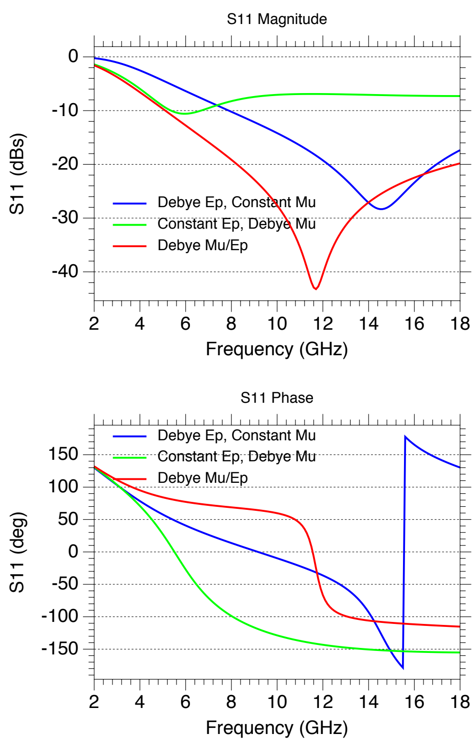


**Figure 7 Magnitude (top) and phase (bottom) for different combinations of Debye model parameters**

To determine the quality of fit for the Debye model inversion, we first obtain the permittivity/permeability data using S11/S21 measurements and conventional inversion of a magnetic sample. These values are fit to a Debye model to give the 'Ideal' fit for an equivalent Debye material. Finally, the S11/S12 data is converted into impedance using equation 2 and fit using the 6 Debye model parameters. The material properties are subsequently extracted using equations 4 and 5, and the materials parameters obtained from the three methods are plotted in Figure 8. From this, the Debye model inversion shows good agreement with S11/S21and idealized Debye fit data.

The Debye model inversion method was found to more accurately fit permeability in thin samples. The reason for this becomes clear from analysis of the reflection equation above. In the limit for thin specimens,

$$\lim_{tk_0 n \to 0} Z = \lim_{tk_0 n \to 0} Z_m tanh(itk_0 n) = Z_m itk_0 n = itk_0 \mu \quad (6)$$

This result implies that impedance will have a large dependence on permeability at lower thicknesses/frequencies. Conversely, the effect of permittivity will be much less, and the fitting for the permittivity parameters will be more challenging. This behavior is consistent with the requirements of a vanishing tangential E-field adjacent to a conductive boundary. On the other hand, as losses within the material increase, whether from higher intrinsic loss or higher frequency/thickness, the impedance of the system converges on to the impedance of the material:

$$Z_m tanh(itk_0 n) = Z_m \frac{\tanh(Im(n)tk_0) - itan(Re(n)tk_0)}{1 - \mathrm{itanh}(Im(n)tk_0)\tan(Re(n)tk_0)} \quad (7)$$

$$\lim_{Im(n)tk_0 \to \infty} Z = Z_m \frac{1 - itan(Re(n)tk_0)}{1 - i\,tan(Re(n)tk_0)} = Z_m \quad (8)$$

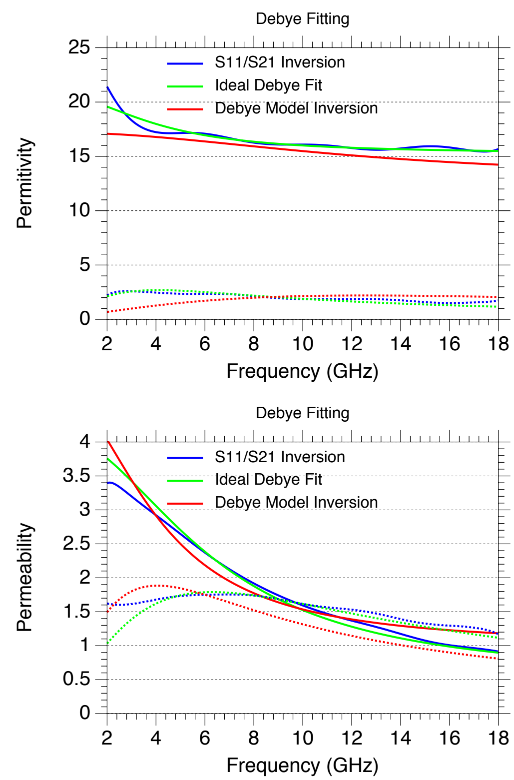


**Figure 8 Extracted properties from i) conventional transmission/reflection inversion ('measured'), ii) Debye model fit to these extracted properties ('modeled') and iii) model-based inversion from metal-backed S11 of magram samples ('fit').**

To demonstrate this change in dependence on permeability to a dependence in $Z_m$, Figure 9 shows simulated data magnetic material at two different thicknesses, generated using eq 2 and constant values for ($\mu = 6 - 0.1i$, $\varepsilon = 10 - 0.6i$), compared with the limit values obtained in equations 6 and 8. At a thickness of 0.5 mm, the low frequency is highly correlated with the permeability, but gradually diverges. In contrast, for a sample with thickness of 10 mm, the data can be observed to oscillate about the material's impedance, before gradually converging on it. It should be noted that the impedance still contains a dependence to permeability built in to the materials impedance. Thus, there does not exist a clear region that has high dependence on solely the permittivity. While thinner samples can generate more accurate fitting of the permeability, very thick samples do not necessarily lead to improved fitting for permittivity. Ideally, the material thickness would be chosen to encompass the low and high frequency approximation regions, as this forces the fitting algorithm to select values that will accurately represent all material properties.

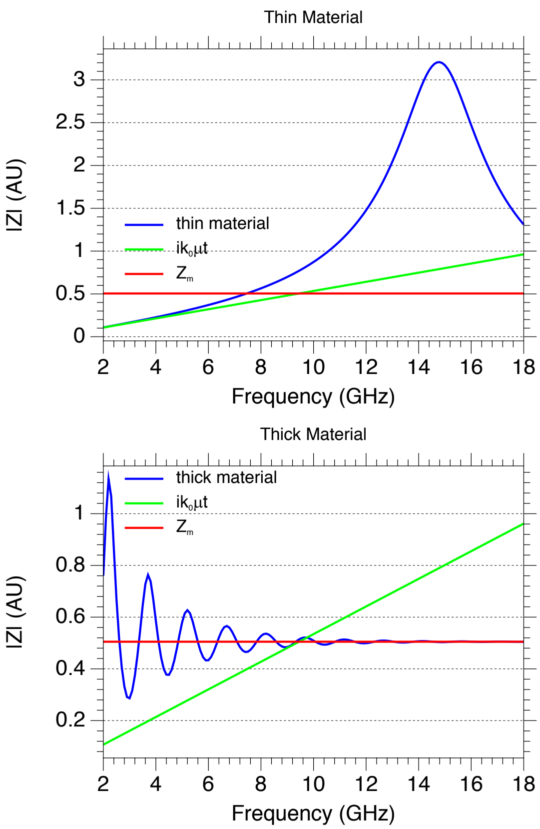


**Figure 9: Simulated impedance for metal-backed absorber with thickness of 0.5 mm (top) and 10 mm (bottom)**

One of the largest limitations to the Debye fitting model is the existence of multiple, degenerate solutions. Ideally, the Debye parameters that lead to the closest fit of the reflection should also lead to the closest fit of the material parameters. In the case of a simulated Debye material, this is indeed true. For actual materials however, the Debye model is not a perfect fit and there is the possibility of an 'inferior' solution that has a superior fit to the reflection. Figure 10 shows that this scenario can occur in the model fitting, leading to less than optimal inversion of the material parameters. While this level of error may be acceptable in some applications, generating more precise matches to the data requires bounds on some of the fitting parameters to reduce the chance for problematic solutions to be selected.

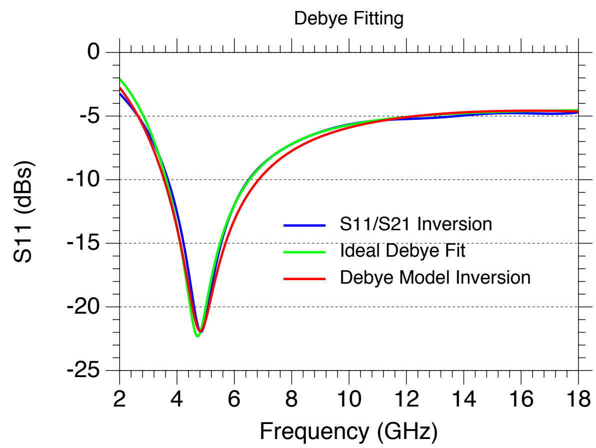


**Figure 10: Comparison between Debye model fitting to permittivity/permeability data and fitting to reflection data.**

## V. Conclusion

This paper presented two methods for determining permittivity and permeability of metal backed samples. Both methods have advantages and disadvantages, and neither should replace traditional transmission/reflection measurements whenever such measurement are possible. With this caveat, the materials parameters extracted by these methods show agreement with measurements based on conventionally inverted transmission/reflection data.

The Debye model method does well if the bandwidth of data encompasses a wide enough range of the material behavior. In addition, including bounds on the fitted parameters minimizes the problem of multiple solutions and improve reproducibility of inversions. In cases where the material under test cannot be assumed to be Debye-like, the two-thickness method provides a better match the materials properties, at the cost of requiring an additional sample to be fabricated.

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