Measuring Complex Permittivity, Permeability, or Sheet Impedance of Materials at Microwave Frequencies with a Free-Space Focused Beam

1. Scope

1.1 This test method covers a procedure for determining relative complex permittivity (relative dielectric constant and loss) and relative magnetic permeability of reciprocal (non-gyromagnetic) solid materials. If the material is nonmagnetic, it is acceptable to use this procedure to measure permittivity only.

1.2 This measurement method is valid over a frequency range of approximately 1 GHz to over 100 GHz. These limits are not exact and depend on the size of the specimen, the design of the lenses, the frequency range of the feed horns, and on the applicable frequency range of the network analyzer used to make measurements. The practical lower frequency is limited by specimen dimension requirements (large specimens at low frequencies). Being a non-resonant method, any number of discrete measurement frequencies may be selected in a measurement band.

1.3 The values stated in SI units are to be regarded as the standard. The values given in parentheses are in English units. The equations shown here assume an $e^{+j\omega t}$ harmonic time convention.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 ISBN 1480092851 Focused Beam Methods: Measuring Microwave Materials in Free Space, J.W. Schultz, 2012

3. Terminology

3.1 For other definitions used in this test method, refer to ISBN 1480092851 Focused Beam Methods.

3.2 *Definitions:*

3.2.1 *Relative complex permittivity (relative complex dielectric constant),* ε_r^*—the proportionality factor that relates the electric field to the electric flux density, and which depends on intrinsic material properties such as molecular polarizability, charge mobility, etc.:

$$\boldsymbol{\varepsilon_r^* = \varepsilon_r' - j\varepsilon_r'' = \frac{\vec{D}}{\varepsilon_0\vec{E}}} \qquad (1)$$

where

ε_0= the permittivity of free space,

$\vec{D}$ = the electric flux density vector, and

$\vec{E}$ = the electric field vector.

3.2.1.1 *Discussion*—In common usage the word "relative" is frequently dropped. The real part of complex relative permittivity (ε_r') is often referred to as simply relative permittivity, permittivity or dielectric constant. The imaginary part of complex relative permittivity (ε_r'') is often referred to as the loss factor. In anisotropic media, permittivity is polarization dependent and described by a three-dimensional tensor.

3.2.1.2 For the purposes of this test method, only one tensor component of the media is measured at a time, and therefore permittivity is a single complex number at each frequency.

3.2.2 *Relative complex permeability,* μ_r^*—the proportionality factor that relates the magnetic flux density to the magnetic field, and which depends on intrinsic material properties such as magnetic moment, domain magnetization, etc.:

$$\mu_r^* = \mu_r' - j\mu_r'' = \frac{\vec{B}}{\mu_0\vec{H}} \qquad (2)$$

where:

μ_0= the permeability of free space,

$\vec{B}$ is the magnetic flux density vector, and

$\vec{H}$ is the magnetic field vector

3.2.2.1 *Discussion*—In common usage the word "relative" is frequently dropped. The real part of complex relative permeability (μ_r') is often referred to as relative permeability or simply permeability. The imaginary part of complex relative permeability (μ_r'') is often referred to as the magnetic loss factor. In anisotropic media, permeability is polarization dependent and described by a three-dimensional tensor.

3.2.2.2 For the purposes of this test method, only one tensor component of the media is measured at a time, and therefore permeability is a single complex number at each frequency.

3.3 *Definitions of Terms Specific to This Standard:*

3.3.1 A list of symbols specific to this test method is given in Annex A1.

3.3.2 *Calibration*—a procedure for inserting well-defined, standard specimens into the focused beam fixture to characterize the measurement system's systematic errors. The effects of the systematic errors are then mathematically removed from the indicated measurements. The calibration also establishes the mathematical reference plane for the measurement test ports.

3.3.2.1 *Discussion*—While modern vector network analyzers have this capability built in, it is also acceptable to make calibration measurements and apply the suitable calibration models as a post-processing step. When both transmission and reflection measurements are used, a two-port calibration is required.

3.3.3 *Network analyzer*—a system that measures the two-port transmission and one-port reflection characteristics of a multiport system in its linear range and at a common input and output frequency.

3.3.3.1 *Discussion*—For the purposes of this standard, this description includes only those systems that have a synthesized signal generator and that measure the complex scattering parameters (both magnitude and phase) in the forward and reverse directions of a two-port network (S_{11}, S_{21}, S_{12}, S_{22}).

3.3.4 *Scattering parameter (S-parameter),* S_{ij},—a complex number consisting of either the reflection or transmission coefficient of a component at a specified set of input and output reference planes with an incident signal on only a single port.

3.3.4.1 *Discussion*—As most commonly used, these coefficients represent the quotient of the complex electric field strength (or voltage) of a reflected or transmitted wave divided by that of an incident wave. The subscripts i and j of a typical coefficient S_{ij} refer to the output and input ports, respectively. For example, the forward transmission coefficient S_{21} is the ratio of the voltage transmitted to port 2 divided by the incident wave voltage from port 1. Similarly, the port 1 reflection coefficient S_{11} is the ratio of the port 1 reflected wave voltage divided by the port 1 incident wave voltage.

3.3.5 *Time domain gate*—a mathematical procedure applied to the calibrated scattering parameter data, which minimizes coherent errors from multipath reflections.

3.3.5.1 *Discussion*—Unwanted reflections from discontinuities in the measurement fixture may add ripple and other systematic errors to the desired specimen scattering parameters. This procedure is equivalent to converting stepped frequency-domain data into the time-domain via Fourier transform, then applying a window function that removes all but the desired signal from the specimen. Many modern network analyzers have this function built in, but it is also acceptable to apply a time-domain gate as a post-processing step.

4. Executive Summary and Significance

4.1 A planar test specimen is placed in a focused beam test fixture, which consists of two horn and lens pairs connected to a network analyzer. This test fixture is used to measure the *S*-parameters of the test specimen, along with appropriate calibration standards. After the data are calibrated, a time-domain gate is applied, and a specified data-reduction algorithm is used to calculate permittivity and permeability. If the material is nonmagnetic, a different algorithm is used to calculate permittivity only.

4.2 Design calculations for microwave and millimeter-wave components require the knowledge of values of complex permittivity and permeability at operating frequencies. This test method is useful for evaluating experimental batch or continuous production materials used in electromagnetic applications. It may be used to determine complex permittivity only (in non-magnetic materials) or both complex permittivity and permeability simultaneously.

5. Restrictions

5.1 Excessive electromagnetic attenuation due to a high loss factor within the test specimen can prevent determination of permittivity and permeability. This is mitigated by reducing the sample thickness so that sample insertion loss is within acceptable accuracy limits of the network analyzer.

5.2 In contrast to coaxial airline and rectangular waveguide methods, there are no restrictions due to air-gaps (i.e. exact fit of a specimen within a fixture is not necessary)

6. Apparatus

6.1 *Experimental Test Fixture*—The test fixture includes two collinearly aligned pairs of feed antennas and lenses, a specimen holder, and a network analyzer, as shown in Fig. 1. The specimen holder is placed at the focal plane between the lenses to ensure a minimum phase taper.

6.2 *Network Analyzer*—The network analyzer must have a 2-port test set that can measure complex transmission and reflection scattering parameters in both directions.

6.3 *Calibration Kit*—To define the measurement reference plane and minimize fixture uncertainties, calibration of the test fixture is required. As a minimum, a response and isolation calibration procedure should be followed. The reflection response standard is a flat metal plate, of no less than 6 mm thickness (so it does not flex), with a machined or polished surface finish. The metal plate is preferably the same size as the specimen and is also used as a transmission isolation standard. The transmission response standard is the empty test fixture (clearsite). A foam absorber tilted approximately 45 degrees from the plane of the specimen holder serves as an optional reflection isolation standard. It is also acceptable to insert an intermediate metal plate at an appropriate tilt angle, which then directs the beam to a well-matched absorber or to a minimum of one meter of free space. It is also sufficient to use the empty focused beam test fixture as the reflection isolation standard. Optionally, a through-reflect-line (TRL) or other calibration procedure that yields similar calibration quality may be used to calibrate the test fixture.

6.4 *Lens Design*:

6.4.1 The test fixture includes two identical focusing lenses, manufactured from a well-characterized dielectric material. The transmit lens accepts radiation from the feed antenna and focuses it onto the test specimen. The receive lens transfers the radiation transmitted through the sample into the receive antenna. The lenses must have a low to moderate dielectric permittivity to minimize multipath reflections, while still providing the necessary focusing power. The lens material permittivity should also be approximately constant across the measurement frequency range.

6.4.2 For this measurement method, the lens specification is defined in terms of two performance characteristics: beam waist and phase taper. Both of these performance characteristics are evaluated at the lens focal point (specimen position). Verification of lens performance characteristics is performed by scanned probe measurements of the focus position or by an inverse synthetic aperture (ISAR) method with a rotated metal plate.

6.4.3 The beam waist, w_0, is the radius of the beam at the focal point where the amplitude is reduced by 8.7 dB relative to the center of the beam. This corresponds to the radius where the field is reduced by a factor of $1/e$, where $e = 2.7183$. In wavelength scaled units, the waist is given by

$$k_0 w_0 = \frac{2\pi w_0}{\lambda_0}. \quad (3)$$

6.4.3.1 The lens is designed to achieve two goals: i) minimize the beam waist so that specimen size can be minimized and ii) approximate a plane wave by minimizing phase taper within the beam. However, a beam waist that is too small results in excessive focusing errors caused by deviation from ideal plane wave behavior. For reasonable accuracy with most materials the beam waist must be no less than $k_0 w_0 = 6$. In non-symmetrical beams where the cross-section is more accurately described as elliptical, the minimum beam waist specification applies to the smallest dimension of the beam.

6.4.3.2 The phase taper of the focus position also affects measurement accuracy and deviation from a plane wave behavior (no phase taper) must be minimized. In the focused beam measurement system, phase taper within the beam waist region must be no more than +/- 15 deg.

7. Test Specimen

7.1 Make the test specimen thick enough to ensure that the phase shift through the specimen is much greater than the phase measurement uncertainty of the network analyzer at the lowest measurement frequency. If a specimen is expected to have low loss, sufficient length is also required to insure accurate determination of the loss factor. For high loss specimens, the specimen thickness cannot be so much that high insertion loss prevents material property inversion. For magnetic specimens, the thickness must be less than $\lambda/2$ at the highest frequency of interest to avoid inversion errors associated with specimen resonances. For dielectric-only materials this $\lambda/2$ restriction does not apply, however the thickness must be less than the focal depth, z_R, of the beam[1],

[1] Defined by the Rayleigh Length in Gaussian Beam theory

$$z_R < \frac{\pi w_0^2}{\lambda}. \quad (4)$$

7.2 Ensure that the lateral specimen dimensions (orthogonal to the thickness direction) are larger than the illuminating beam. The amplitude of the incident beam must be -20 dB or less at the edge of the sample, relative to the beam center.

7.3 Keep thickness variations of the specimen as small as is practicable. Measure the thickness of the specimen at a minimum of five different locations and include specimen dimensions and uncertainties in the report. The thickness measurement locations must be at or near the illumination peak from the focused beam. Measurement of thicknesses at the edge(s) of the specimen are insufficient.

8. Preparation of Apparatus

8.1 *Inspect Network Analyzer Test Ports*—Insure that the test ports are in good working condition. Refer to network analyzer manufacturer's documentation to provide connector specifications.

8.2 *Flexing and Tightening Cables and Connectors*—Cable flexing and improperly tightened connectors introduce phase and magnitude errors into *S*-parameter data. For this reason, bend the test cables as little as possible, and under no circumstances bend the test cables smaller than the manufacturer's minimum recommended radius. Use a torque wrench with the manufacturer's recommended torque to tighten connectors.

8.3 *System Alignment*—The feed horns, lenses, and specimen holder must be aligned so that they are all centered along a common line. Additionally, the planes that define the aperture of horns, lenses and specimen holder must all be aligned orthogonally to that common line. Alignment is established geometrically (through careful positioning of each of these elements), and then refined electromagnetically. There are two procedures for electromagnetic refinement of system alignment.

8.3.1 The first alignment refinement moves one horn/lens pair with respect to the other while monitoring the transmission (S_{21}) with a network analyzer. The maximum transmission occurs when the two halves of the focused beam system are aligned. If the two sides of the focused beam system are built onto a common rail, then this step is unnecessary.

8.3.2 The second electromagnetic alignment rotates the specimen holder with respect to the system. A reflective metal plate is mounted in the specimen holder and it is rotated while monitoring the reflection (S_{11}) with a network analyzer. The maximum reflection occurs when the specimen holder is parallel to the planes of the lens and horn apertures.

8.4 *Network Analyzer Setup*:

8.4.1 Refer to manufacturer documentation for minimum warm-up period for network analyzer.

8.4.2 Use the network analyzer in stepped frequency mode. Set the start frequency and stop frequency as desired. Set the number of measurement points to ensure an unambiguous range of at least twice the path length between the two network analyzer ports. Ignoring the effects of dielectric in the RF cables, the unambiguous range can be estimated by $R_{unambiguous} = cN/(f_{max} - f_{min})$, where N is the number of frequency steps, c is the speed of light, and $f_{min/max}$ are the measured frequency limits. In practical terms, the use of 1801 points over a frequency span of 2 to 20 GHz corresponds to an unambiguous range of ~30 meters (~100 feet), which is sufficient for this method.

8.4.3 Set the network analyzer's variable intermediate frequency (IF) bandwidth to 1000 Hz or less.

9. Procedure

9.1 The test procedure includes calibration and a series of specimen measurements.

9.2 *Calibration*:

9.2.1 Two calibration standards are required: a metal plate that is smooth and flat on both sides, and a clear site (no specimen). A flat broadband foam absorber is optional for the reflection isolation standard.

9.2.2 Use the following procedure to obtain the response (R_{ij}) and isolation (I_{ij}) calibration parameters for all four scattering parameters (S_{11} S_{22} S_{12} and S_{21}):

9.2.2.1 Measure S_{21} and S_{12} (i.e. thru or clear site) with no specimen to obtain transmission response coefficients (R_{21} and R_{12}).

9.2.2.2 Insert the metal plate standard with known thickness t_m and measure S_{11} and S_{22} (R_{11} and R_{22}) *without moving the plate* to obtain reflection response coefficients in both directions.

9.2.2.3 Leaving the plate in place, measure S_{21} and S_{12} to obtain transmission isolation coefficients (I_{21} and I_{12})

9.2.2.4 Remove the metal plate and obtain reflection isolation coefficients (I_{22} and I_{11}). Optionally, insert the broadband foam absorber standard, offset from the specimen position as shown in FIG. 2, before measuring each isolation coefficient.

9.2.2.5 If using the optional absorber, tilt the absorber at 30-45 degrees so that residual energy reflected from absorber is directed laterally away from the focused beam system and measure reflection isolation. Place the absorber as far back as possible (i.e. against the far lens) for each side before collecting the corresponding isolation data.

9.2.2.6 For dielectric-only specimens (relative magnetic permeability is known a priori to be 1), it is sufficient to calibrate with steps 9.2.2.1 and 9.2.2.3, and the reflection calibration steps may be omitted.

9.2.3 *Calibration Frequency*—For a series of specimen measurements, repeat the calibration procedure at least every 90 minutes to check for the effects of temperature or system drift. For any given measurement sequence, the calibration sequence must be conducted at least twice (e.g. before and after the measurements) to check for the effects of temperature or system drift.

9.2.4 *Measurement Validation*—For every measurement sequence, a validation specimen must also be measured to check for proper system operation. This specimen must be a known material that is measured at least once every day that the system is in use. This provides validation for the measurements of that day and also tracks the accuracy of the measurement system over time.

9.3 *Specimen Measurement*:

9.3.1 Measure and record the thickness of the specimen with a micrometer or other precision instrument. (This step may be performed at any time)

9.3.2 For 2-port measurements, insert the specimen into the sample holder and collect all four scattering parameters with the network analyzer without moving the specimen. The inversion algorithms in this procedure do not depend on the specimen location, so special care in positioning the specimen is not necessary. For typical focused beam fixtures, the sample front face should be within a few millimeters of the reference plane established by the metal plate calibration to ensure it is within the focal depth, z_R, of the beam.

9.3.3 For 1-port measurements (i.e. dielectric or resistive sheets), it is sufficient to measure the transmission coefficient in one direction and the reflection scattering parameter measurements may be omitted.

9.3.4 Remove the specimen and if additional specimens exist, they can be measured by repeating 9.3.2 or 9.3.3

9.3.5 If 90 minutes or more has lapsed since the calibration, renew the calibration.

9.3.6 Apply the calibration data to the specimen measurements, apply time domain gating, and calculate the intrinsic properties of the specimen(s) from the calibrated scattering parameters, as described in the following Calculation section.

10. Calculation

10.1 The calibrated scattering parameters for a given specimen are obtained from the calibration parameters by,

$$\boldsymbol{S_{ij}^{calibrated}} = \frac{\boldsymbol{S_{ij}^{measured} - I_{ij}}}{\boldsymbol{R_{ij} - I_{ij}}} \qquad (5)$$

where ij is the set of scattering parameters (11, 12, 21, 22).

10.2 Time domain gating must be applied to the calibrated scattering parameters to further reduce errors. For 2-20 GHz measurements, a 1 ns width gate is typical. For other frequency ranges or other specimen types, different time domain settings will be appropriate. Check that the gate window width fully encompasses the measured specimen signal by viewing the frequency data in time domain. More resonant specimens, such as specimens thicker than $\lambda/2$ within the measurement band, require longer gate widths.

10.3 The selection of data reduction algorithm for calculation of material characteristics depends on whether or not the sample is magnetic. Two calculation procedures are provided in this standard: four-parameter and one-parameter. The four-parameter procedure requires four different network scattering parameters (S_{11}, S_{22}, S_{21}, S_{12}) and provides both μ_r^*, and ε_r^*. The one-parameter procedure requires only one network scattering parameter (S_{21}) and provides ε_r^* for a non-magnetic ($\mu_r^* = 1$) material specimen.

10.4 *Iterative four-parameter algorithm*—The iterative algorithms begin with initial estimates at all frequencies for permittivity (and permeability). The algorithm then refines the initial estimates with an iterative technique. There are often multiple solutions to these equations, and an initial estimate is necessary to select the proper root. Suitability of the initial estimates are determined by the stability of permittivity and permeability results, or by comparison with other measurement methods. If the initial estimates start the iterative calculation on the wrong root, the calculated results tend to have poor convergence and may jump to another root when plotted as a function of frequency. Improper solutions may also violate energy conservation by having negative loss. The equations relating the scattering parameters to specimen thickness, permittivity and permeability are as follows:[2]

$$\boldsymbol{S_{11}^{cal}S_{22}^{cal}e^{-2\gamma_0(t_s-t_m)}-S_{11}^{cal}S_{22}^{cal}e^{-2\gamma_0(t_s)}} = \frac{\Gamma^2-\mathrm{T}^2}{1-\Gamma^2\mathrm{T}^2} \quad (6)$$

$$\boldsymbol{e^{-\gamma_0 t_s}\frac{S_{21}^{cal}+S_{12}^{cal}}{2}} = \frac{\mathrm{T}(1-\Gamma^2)}{1-\Gamma^2\mathrm{T}^2} \quad (7)$$

where $\boldsymbol{\Gamma} = \frac{\mu_r^*-\sqrt{\mu_r^*\varepsilon_r^*}}{\mu_r^*+\sqrt{\mu_r^*\varepsilon_r^*}}$, $\mathbf{T} = \boldsymbol{e^{-\gamma t_s}}$, $\boldsymbol{\gamma_0} = \sqrt{-\boldsymbol{k_0^2}}$, $\boldsymbol{\gamma} = \sqrt{-\boldsymbol{\mu_r^*\varepsilon_r^*k_0^2}}$, and k_0 is the wavenumber in air (= $2\pi/\lambda_0$).

10.4.1 The 'cal' superscript designates the use of already calibrated scattering parameters, and the algorithm must also account for the transmission line displaced with the metal calibration plate by including the calibration plate thickness, t_m as well as the specimen thickness, t_s. These equations are numerically solved via Newton's iteration algorithm[3]. For complex computation, rearrange equations (6) and (7) into zero-valued functions and differentiate with respect to $\boldsymbol{\varepsilon_r^*}$, and $\boldsymbol{\mu_r^*}$ to generate a system of two equations. In matrix form this is represented by:

$$\begin{bmatrix} \frac{\partial F}{\partial \varepsilon_r^*} & \frac{\partial F}{\partial \mu_r^*} \\ \frac{\partial G}{\partial \varepsilon_r^*} & \frac{\partial G}{\partial \mu_r^*} \end{bmatrix} \cdot \begin{bmatrix} \Delta\varepsilon_r^* \\ \Delta\mu_r^* \end{bmatrix} = \begin{bmatrix} F \\ G \end{bmatrix} \quad (8)$$

$$\text{where } F = (1-\Gamma^2\mathrm{T}^2)\left(S_{11}^{cal}S_{22}^{cal}e^{-2\gamma_0(t_s-t_m)}-S_{11}^{cal}S_{22}^{cal}e^{-2\gamma_0(t_s)}\right) - (\Gamma^2-\mathrm{T}^2) \quad (9)$$

$$\text{and } G = (1-\Gamma^2\mathrm{T}^2)\left(e^{-\gamma_0 t_s}\frac{S_{21}^{cal}+S_{12}^{cal}}{2}\right) - \mathrm{T}(1-\Gamma^2) \quad (10)$$

and where the two-by-two matrix is the Jacobian of the system of two complex equations.

10.4.2 In matrix notation the discretized system of equations can be represented as follows:

$$\boldsymbol{J\Delta X} = \boldsymbol{Y} \quad (11)$$

where the vector X contains the estimated values of permittivity and permeability, and the vector Y contains the equations (9) and (10).

10.4.3 Solving for ΔX gives

$$\boldsymbol{\Delta X} = \boldsymbol{J^{-1}Y}. \quad (12)$$

10.4.4 To solve for permittivity and permeability, start with an initial estimate of X and calculate ΔX. The functional iteration procedure is then:

$$\boldsymbol{X^{new}} = \boldsymbol{X^{old}} - \boldsymbol{J^{-1}Y} \quad (13)$$

10.4.5 Repeat this until $X^{new} \approx X^{old}$, at which point the converged values of permittivity and permeability have been found.

10.5 *Iterative one-parameter algorithm*— When the specimen is known to be non-magnetic, then permittivity can be inverted iteratively with just the S_{21} scattering parameter data. The procedure iteratively solves the following equations for permittivity,

$$e^{-\gamma_0 L}S_{21}^{cal} = \frac{\mathrm{T}(1-\Gamma^2)}{1-\Gamma^2\mathrm{T}^2} \quad (14)$$

$$\text{where } \Gamma = \frac{\gamma_0-\gamma_1}{\gamma_0+\gamma_1} \text{ and } \mathrm{T} = e^{-\gamma_1 L}, \quad (15)$$

where $\gamma_0 = \sqrt{-k_0^2}$ and $\gamma_1 = \sqrt{-\varepsilon_r^* k_0^2}$ and k_0 is the wavenumber in air (= $2\pi/\lambda_0$). These equations are solved in a manner similar to the four-parameter algorithm.

---

[2]Baker-Jarvis, J. R., Janezic, M. D., Grosvenor, J. H., and Geyer, R. G., "Transmission/Reflection and Short-Circuit Line Methods for Measuring Permittivity and Permeability," *NIST Technical Note 1355*, May 1992.

[3] W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes 3rd Edition: The Art of Scientific Computing, Cambridge University Press, (2007)

10.6 *Resistive Sheets*— When the specimen consists of a non-magnetic resistive sheet, the sheet impedance in ohms/square can be calculated from just the S_{21} scattering parameter data. The relationship between sheet impedance, Z, and S_{21} is given by,

$$Z = \frac{Z_0 S_{21}}{2(1-S_{21})}, \qquad (16)$$

where Z_0 is the impedance of free space.

11. Report

11.1 Report the following information:

11.1.1 Operator name, time and date of measurement,
11.1.2 Focused beam system information,
11.1.3 Network analyzer setting such as start and stop frequencies, number of points, averaging factor / IF bandwidth,
11.1.4 Algorithm method,
11.1.5 Calculated values of the permittivity and permeability of the verification specimen at each measurement frequency,
11.1.6 Test specimen identification and origin,
11.1.7 Specimen dimensions and uncertainties used in data reduction,
11.1.8 Specimen flatness, and
11.1.9 Calculated values of permittivity, permeability, or impedance for the test specimen at each measurement frequency.

12. Precision and Bias

12.1 *Precision*—It is not practicable to specify the precision of the procedure in this test method for measuring permittivity and permeability because of the multiple variables that influence precision. In specific measurements, it is possible to estimate measurement precision by estimating the uncertainties of the measured scattering parameters and specimen dimensions, and then applying a differential analysis to the given equations.[3] Example uncertainty calculations are in ISBN 1480092851 Focused Beam Methods.

12.2 The sources of error in permeability and permittivity measurement include:

12.2.1.1 Errors in measuring the magnitude and phase of the scattering parameters,
12.2.1.2 Uncertainty in the specimen thickness,
12.2.1.3 Line losses and connector mismatch,
12.2.1.4 Focusing errors[4], and
12.2.1.5 Alignment errors.

12.3 *Bias*—Bias of the procedure in this test method can occur if the sample is over-illuminated. Over-illumination results in edge diffraction and increased transmission around the sample. This bias is minimized when the power density of the illuminating beam at the sample edge is no more than -20 dB relative to the peak. Thus for a typical, $k_0 w_0 = 8$ beam, the lateral dimensions of the specimen should be no smaller than 6 wavelengths across.

[4] L.E.R. Petersson, G.S. Smith, "An estimate of the error caused by the plane-wave approximation in free-space dielectric measurement systems," IEEE Trans. AP, 50(6), 878-887, June 2002

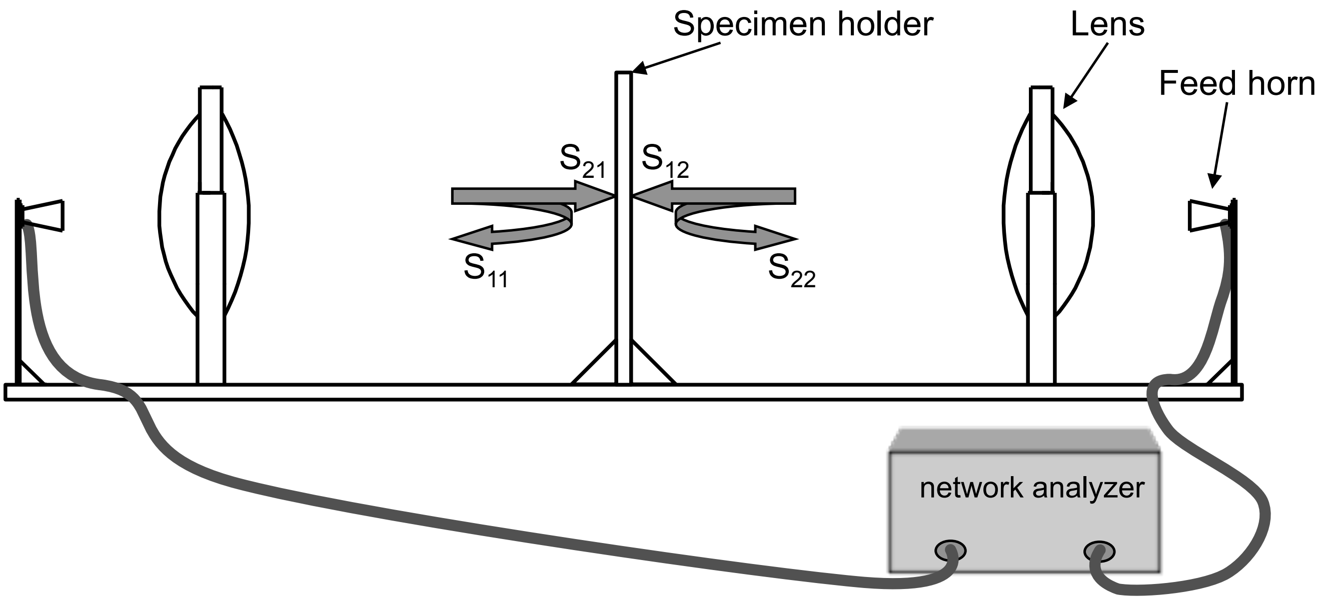

FIG. 1 Diagram of Experimental Fixture

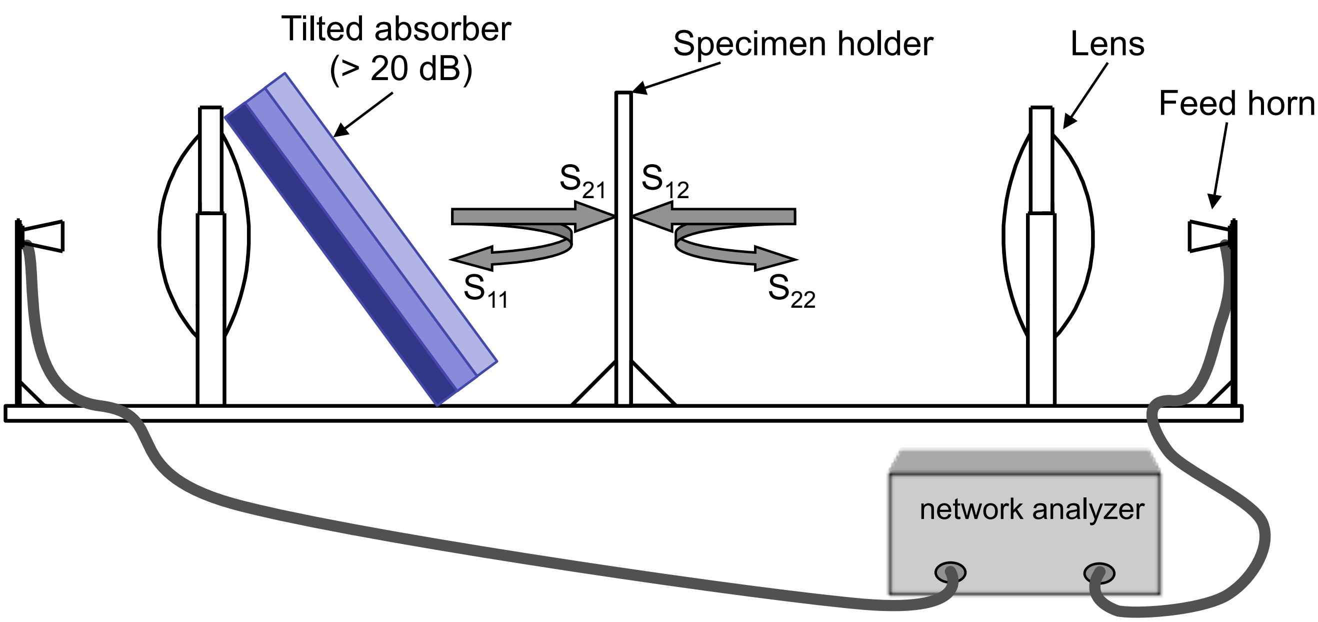

FIG. 2 Sketch of focused beam showing placement of *optional* absorber standard ('matched load') for isolation calibration of S_{22}. The absorber is placed on the opposite side for isolation calibration of S_{11}.

13. ANNEXES

A1. LIST OF IMPORTANT SYMBOLS

$j = \sqrt{-1}$	The complex constant
$c_0 = 2.9979 \times 10^8$	Speed of light in free space (m/s)
$\varepsilon_0 = 8.854 \times 10^{-12}$	Permittivity of free space (Farads/m)
$\mu_0 = 4\pi \times 10^{-7}$	Permeability of free space (Henrys/m)
f	Measurement frequency (Hz)
$\omega = 2\pi f$	Radian frequency (rad/sec)
$\lambda_0 = \frac{c_0}{f}$	Wavelength in free space (m)
$\varepsilon_r^* = \varepsilon_r^{'} - j\varepsilon_r^{''}$	Relative complex permittivity of specimen
$\mu_r^* = \mu_r^{'} - j\mu_r^{''}$	Relative complex permeability of specimen
S_{ij}	Scattering coefficient from Port j into Port i
R_{ij}	Response calibration scattering coefficient from Port j into Port i
I_{ij}	Isolation calibration scattering coefficient from Port j into Port i
w_o	Waist of focused beam (at focal point) (m)
t_s	Specimen thickness (m)
t_m	Metal calibration plate thickness (m)
$k_o = \frac{2\pi}{\lambda_0}$	Wavenumber in free space (rad/m)
$\gamma_0 = \sqrt{-k_0^2}$	Propagation constant in free space (rad/m)
$\gamma = \sqrt{-\mu_r^* \varepsilon_r^* k_0^2}$	Propagation constant in specimen (rad/m)
$z_R = \frac{\pi w_0^2}{\lambda}$	Focal depth of focused beam (m)
Γ	Reflection coefficient at specimen/air interface
T	Transmission coefficient through specimen

CTG takes no position respecting the validity of any patent rights asserted in connection with any item mentioned in this standard. Users of this standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, are entirely their own responsibility.

This standard is subject to revision at any time by CTG.